# AN EXPERIMENTAL STUDY OF ECONOMICAL LEARNING

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## I

*Introductory.* The complaint that high school students are unable to perform fundamental computations with speed and accuracy is emphatic and of long standing. The time devoted to arithmetical instruction in the elementary school is disproportionate to the results obtained; and this will remain true while methods of teaching remain unsupported by methods of learning. What is true of the elementary school is only less true of the high school. It is not an exaggerated statement to affirm that the work in arithmetic could be accomplished in two-thirds of the time now required, if more economical methods of learning were employed and an ideal of speed and accuracy developed.

Meuman[1] emphasizes the necessity of supplementing methods of teaching by methods of learning: "This question (of economical learning) becomes the more pressing in modern times because our courses of study, in their attempt to comply with the increasing requirements of practical life, are becoming more and more exacting in the demands which they make upon the memory tasks of school children." Curriculum building is all in the direction of addition of subject matter without hint of compensation by subtraction. If over-burdening of the child is to be avoided it would seem that either subtraction must keep pace with this addition or the rate of learning must be increased by economy of method.

## II

*The Problem.* The primary purpose of this study is to determine quantitatively the superiority of a method which eliminates unessential processes in the four fundamental operations. Otherwise stated, the purpose is to determine the loss in speed and accuracy accompanying the traditional methods of teaching the four fundamental operations. The two methods will be referred to in this study as the 'traditional' and 'economical' methods.

[1] The Psychology of Learning, p. xiv.

The solution of the problem in the margin may serve as an illustration of the waste and fatigue in the prevailing method. When each digit and connecting terms are definitely expressed no less than seventy-one articulations are required. This number is increased by sixteen when the carrying process is formally expressed. Various abbreviations of this procedure are employed but it is exceptional to find a first year high school pupil who, by merely naming results, employs as few as seventeen in the solution.

| |
|---|
| 245 |
| 987 |
| 621 |
| 876 |
| 912 |
| 543 |
| 4,184 |

The difference between the eighty-seven and seventeen expressions represents so much waste and fatigue. Other things being equal, the speed attained should increase with an approach to the smaller number of words. Special attention to the training of the pupils in group addition would enable them further to reduce this number to a minimum of ten words or less. In this study pupils were not trained in group additions for the reason that it would obscure the present problem and for the additional reason that the manner of grouping would not be uniform for all pupils. The traditional and the economical methods would both be proportionally shortened by application of group addition.

Experiments by Pintner,[2] Huey,[3] and Dearborn,[4] show conclusively that articulation in silent reading hinders both rapidity and efficiency of thought getting. As in reading, so in arithmetic, the direct linking of visual forms of digits and groups of digits, without the cumbersome and fatiguing audito-motorizing mechanism, leads to greater rapidity and accuracy in arithmetical computation.

## III

*Procedure.* The Courtis Tests in Arithmetic were given to seventy-six first year high school pupils. From this number sixty-four were selected and divided into two equal groups, each group having approximately the same average initial speed and accuracy. Group T was drilled in the traditional method and Group E was trained in the economical method. Both groups were given identical problems in each of the four fundamental operations. All objective conditions were equalized as completely as possible.

Each group was urged to solve the entire list of twenty-eight problems as rapidly and accurately as possible. The

[2] 'Silent Speech,' *Pintner, Psychological Review,* 1913.
[3] 'The Psychology and Pedagogy of Reading,' *Huey,* 1900.
[4] 'The Psychology of Reading,' *Dearborn, Archives of Philosophy.* Vol. 4, 1906.

instruction given Group E differed from that given Group T only in the added direction that they must 'think results only.' The first column in the illustrative problem given above, therefore, would be read 5, 11, 12, 19, 24, and the second column 6, 7, 14, 16, 24, 28, 8. It should be stated that this group was frequently reminded to "think in terms of results only," since there was evidence that old habits functioned persistently.

There were eight work periods for each group. To equalize the amount of practice for the two groups, and for all the individuals, the work-limit[5] method was chosen in preference to the time-limit method. During a given work period each pupil solved seven problems in each of the four fundamental operations. The two groups practiced at different periods; Group T at the beginning, Group E at the close of the session, thus giving the former group what is commonly regarded as the more favorable hour.

IV

*Experimental Material.* It was especially desirable to make all the problems of the various sets of equal difficulty. To this end, the nine digits were made to recur with the same frequency. It is probable that such usage of the digits secures a desirable distribution of fatigue to the problems of a given set and also makes possible a uniform basis of comparison of work accomplished in the successive work periods.

Each problem in addition consisted of six three-digit numbers. The problems in subtraction involved two nine-digit numbers. In multiplication, the multiplier was a one-digit number in order to avoid the increase of addition processes by adding several partial products. In division, the divisor was a single digit so as to confine the processes as narrowly as possible to division only. The digits one and two were not used as multipliers or divisors.

V

*Results.* To provide a standard measurement of results, the Courtis Tests were given again after the last work period.

[5] To enable pupils to record the time required for the completion of the addition, subtraction, multiplication and division lists respectively, the time in seconds was recorded by the instructor every ten seconds in large figures on the blackboard. As soon as each set of problems was finished the last recorded time was written by the pupil in a place designated at the end of the set. This plan consumed none of the pupil's time in computing the number of seconds required for the several sets and removed a source of possible error. The time required for each of the four sets of problems was then obtained by the experimenter from the data sheets.

The relative efficiency of the two groups at the beginning of the experiment is shown in Tables I and II.

TABLE I

AVERAGES FOR COURTIS TESTS IN THE FOUR FUNDAMENTAL OPERATIONS

| Group | Initial Test | | | | Final Test | | | | Improvements in per cent | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Attempts | | Rights | | Attempts | | Rights | | | |
| | Med. | Var. | Med. | Var. | Med. | Var. | Med. | Var. | Att. | Rts. |
| T..... | 12.9 | .24 | 12.0 | .24 | 14.0 | .20 | 11.7 | .32 | 8.5 | -2.5 |
| E..... | 12.5 | .25 | 11.0 | .28 | 16.8 | .19 | 14.4 | .24 | 34.4 | 30.9 |
| Difference..... | .4 | .01 | 1.0 | .04 | 2.8 | .01 | 2.7 | .08 | 25.9 | 33.4 |
| Superiority of E in (%) | -3.1 | -4.2 | - 8.3 | -16.7 | 20.0 | 5.00 | 24.8 | 25.00 | 23.1 | 33.1 |

The former shows averages for initial and final Courtis tests with a comparison of efficiency and improvement of the two groups. The median number of problems attempted by Group T in the initial Courtis test was .4 higher than those by Group E. The median number of problems correctly solved by Group T was also higher by 1. The variability of Group T was less than that of Group E. This gives Group T an initial superiority over Group E. Table II gives a comparison of the average time required by the two groups to complete the seven problems in each of the four fundamental operations during the eight work periods. This table does not indicate an initial advantage for Group T, the average total time of Group E being five seconds less than that of Group T.

TABLE II

COMPARISON OF GROUP AVERAGES IN SECONDS
(With penalty added for errors)

| Work Periods | Group T | | | | | Group E | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | + | — | × | ÷ | Total | + | — | × | ÷ | Total |
| 1 | 172 | 199 | 211 | 218 | 800 | 169 | 213 | 200 | 213 | 795 |
| 2 | 141 | 192 | 193 | 172 | 698 | 169 | 193 | 195 | 177 | 734 |
| 3 | 162 | 148 | 224 | 219 | 753 | 183 | 168 | 228 | 246 | 825 |
| 4 | 138 | 134 | 186 | 191 | 649 | 161 | 152 | 205 | 190 | 708 |
| 5 | 161 | 196 | 188 | 177 | 722 | 164 | 175 | 172 | 172 | 683 |
| 6 | 154 | 182 | 199 | 192 | 727 | 144 | 141 | 177 | 173 | 635 |
| 7 | 140 | 187 | 185 | 181 | 693 | 129 | 173 | 173 | 179 | 654 |
| 8 | 144 | 186 | 183 | 172 | 685 | 125 | 163 | 163 | 148 | 599 |

The ordinates of the curves in Fig. 1 represent group averages in seconds for the eight work periods in the four fundamental operations. Each point of the curve, therefore, indicates the average number of seconds required by the group to solve the seven problems.

It was impossible, of course, to equate perfectly the intelligence of the two groups. In order to test more effectively the economic method, the values for the first work period of the various curves shown in Figs. 1, 2 and 3 were derived by giving both groups practice in the method which applied to Group T only in subsequent work periods. Obviously, this gives Group T a handicap of one work period in addition to the practice afforded by the initial Courtis test. A glance at the curves b and c, Fig. 4, indicates that the above mentioned handicap was unnecessary, since the curve for Group T already shows a superiority over Group E, excepting at point Att. in multiplication and at points, Att. and Rt. in division.

The handicap operates to the disadvantage of Group E in two ways,—it facilitates the efficiency of Group T on the one hand and accentuates the disadvantageous habits in Group E on the other.

The data in Tables III and III (a) for the initial Courtis Tests show that the median number of problems attempted by Group T was 1.2 higher than Group E in addition, .7 higher in subtraction, .3 lower in multiplication and .2 lower in division, giving Group T a total initial advantage of 1.4 "attempts." The median number of problems correctly solved by Group T in the initial Courtis Test was 1.6 higher than in Group E in addition, .7 in subtraction and 2.3 in multiplication and lower by .6 in division, giving Group T a total initial advantage of 4 "rights." These tables also show the initial variability in the number of problems attempted by Group E to be greater by 5% in addition, 5% in subtraction, equal variability in multiplication and 5% less in division,—an average initial disadvantage of 1.25% for the four fundamental operations. The variability of problems correctly solved by Group E was initially greater by 8% in addition, 9% in subtraction, 10% in multiplication and less by 9% in division,—giving Group T an average initial advantage of 4.5%.

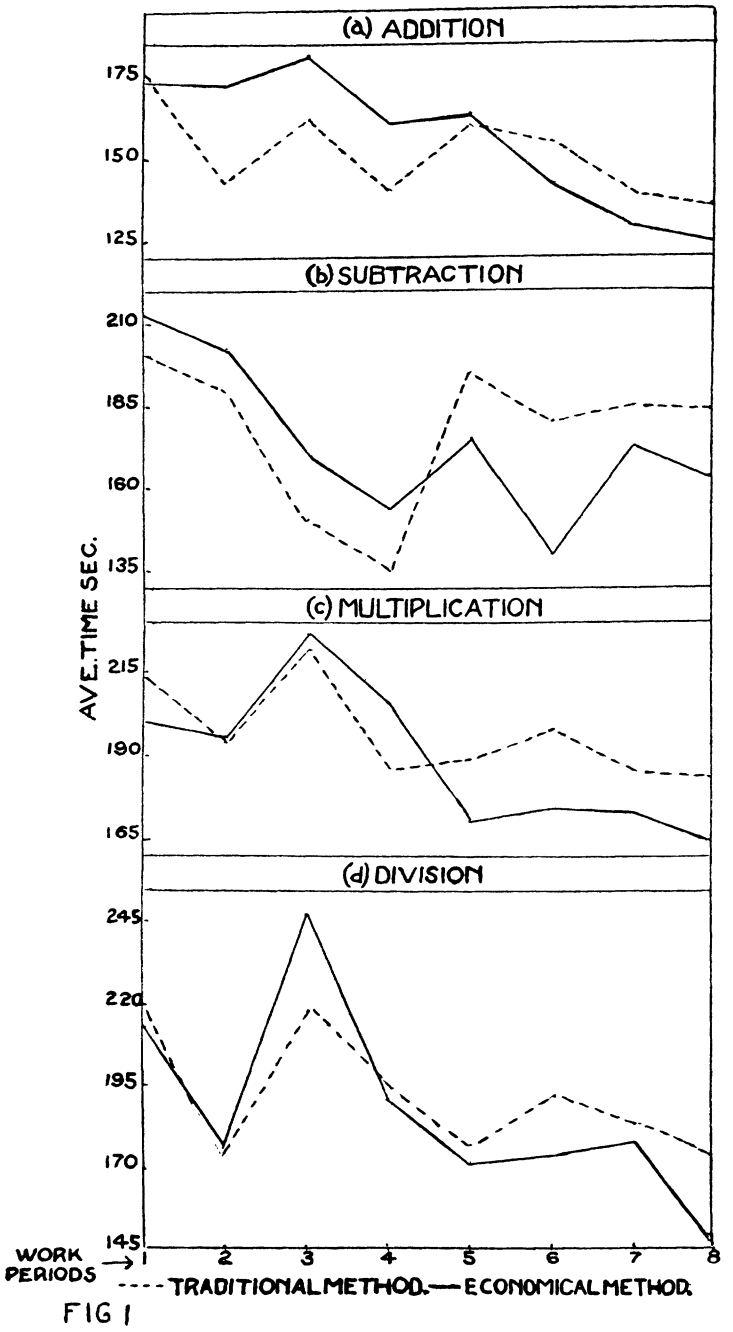
(a) ADDITION
175
150
125
(b) SUBTRACTION
210
185
160
135
AVE. TIME SEC.
(c) MULTIPLICATION
215
190
165
(d) DIVISION
245
220
195
170
145
WORK PERIODS →
1
2
3
4
5
6
7
8
---- TRADITIONAL METHOD. — ECONOMICAL METHOD.

FIG 1

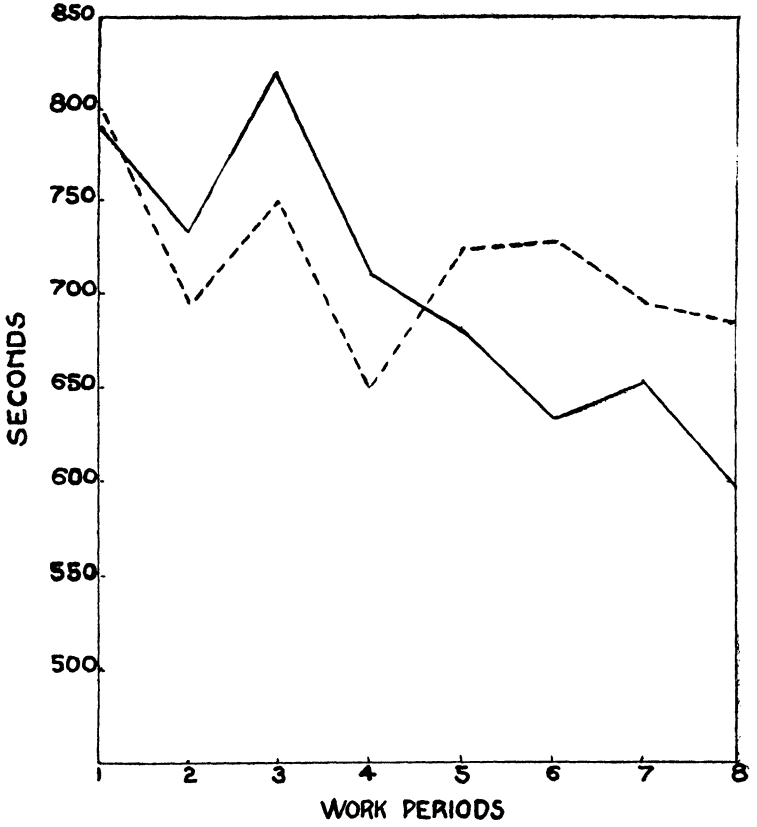


FIG. 2 LEARNING CURVE, showing total time each work period
GROUP E= ——— GROUP T= - - - - -

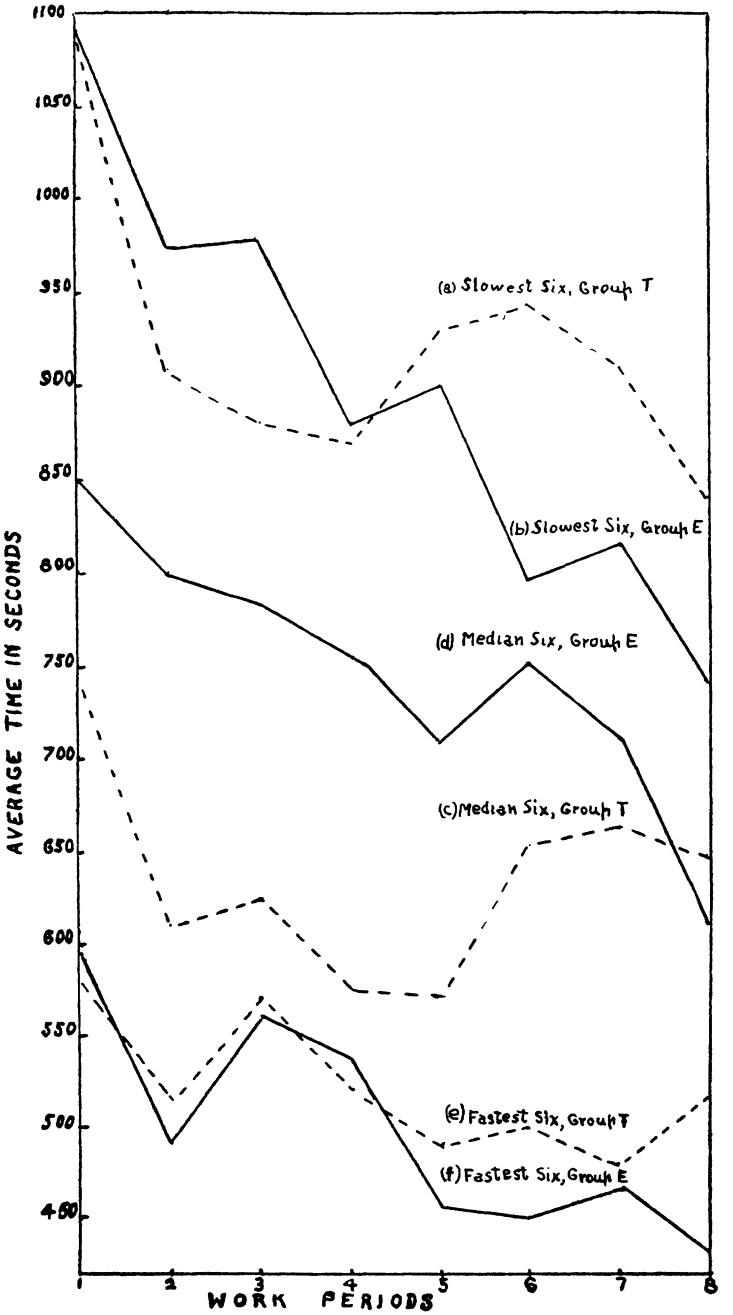


FIG. 3. LEARNING CURVES
FOUR FUNDAMENTAL OPERATIONS COMBINED

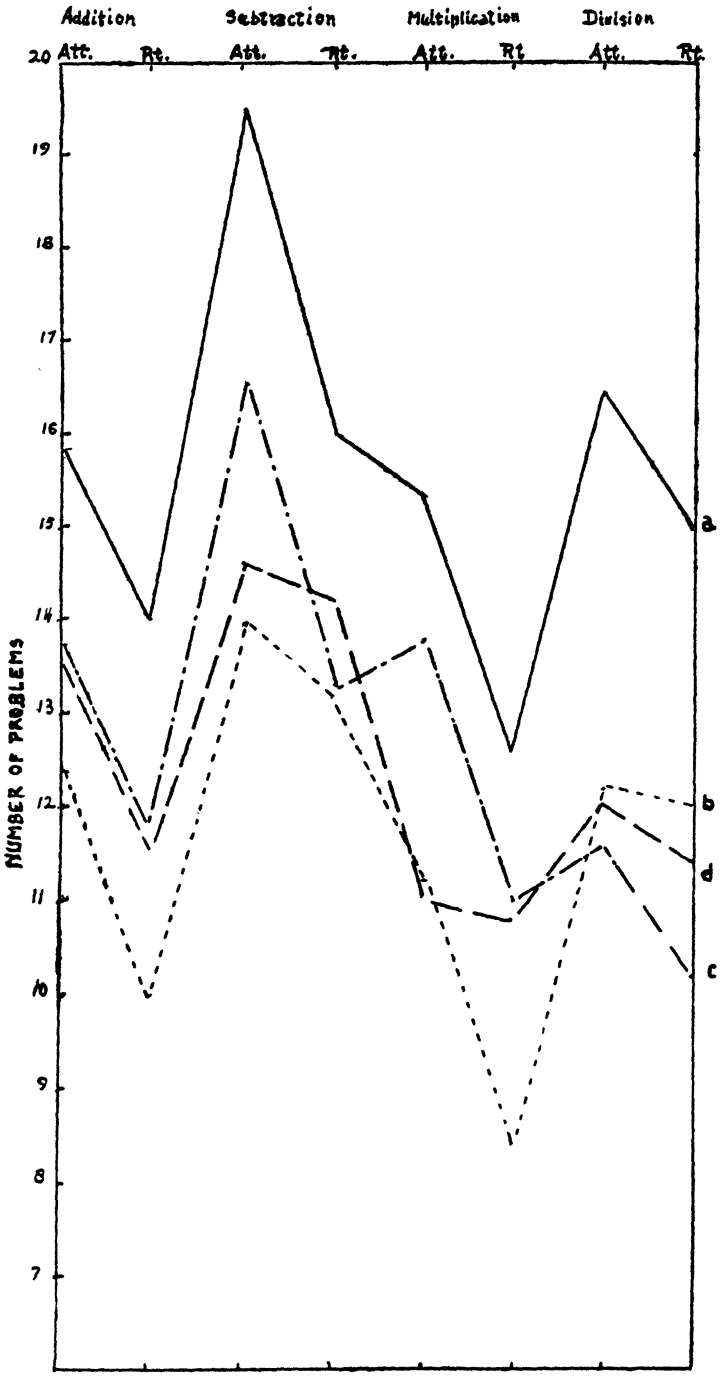


FIG. 4 INITIAL AND FINAL COURTIS TESTS

Att., Number of Problems Attempted. Rt. Number of Problems Right.

——— a, Final Test, Group E - - - - - b, Initial Test, Group E.

—·— · C, Final Test, Group T. — — d, Initial Test, Group T

## TABLE III

### Initial Courtis Test

Group T

| Addition | | | | | Subtraction | | | | | Multiplication | | | | | Division | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Atts. | | | Rts. | | Atts. | | | Rts. | | Atts. | | | Rts. | | Atts. | | | Rts. | |
| Frequency | Deviation | Score | Frequency | Deviation | Frequency | Deviation | Score | Frequency | Deviation | Frequency | Deviation | Score | Frequency | Deviation | Frequency | Deviation | Score | Frequency | Deviation |
| | | 24 | | | | | 24 | | | | | 24 | | | | | 24 | | |
| 2 | | 23 | | | 1 | | 23 | | | | | 23 | | | | | 23 | | |
| | | 22 | | | | | 22 | | | | | 22 | | | 1 | | 22 | 1 | |
| | | 21 | | | 1 | | 21 | 1 | | | | 21 | | | | | 21 | | |
| 2 | | 20 | 3 | | 1 | | 20 | 1 | | | | 20 | | | | | 20 | | |
| 1 | | 19 | | | | | 19 | 1 | | | | 19 | | | 1 | | 19 | 1 | |
| | | 18 | | | 2 | | 18 | | | 1 | | 18 | | | 2 | | 18 | 2 | |
| 1 | | 17 | | | 4 | | 17 | 1 | | 1 | | 17 | | | 1 | | 17 | 1 | |
| 3 | | 16 | 2 | | 4 | 7 | 16 | 7 | 10 | | | 16 | 1 | | 1 | | 16 | 1 | |
| 1 | 5 | 15 | 2 | | | 4 | 15 | 4 | 9 | 3 | | 15 | 2 | | 2 | 3 | 15 | | |
| 3 | 6 | 14 | 2 | 5 | 6 | 6 | 14 | 2 | 2 | 3 | | 14 | | | 1 | 3 | 14 | 2 | 4 |
| 6 | 6 | 13 | 3 | 4 | 4 | | 13 | 5 | | 5 | 7 | 13 | 1 | | 3 | 8 | 13 | 1 | 2 |
| 3 | | 12 | 1 | 6 | 3 | | 12 | 3 | | 3 | 5 | 12 | 4 | 6 | 4 | 4 | 12 | 4 | 7 |
| 4 | | 11 | 6 | 6 | 1 | | 11 | 2 | | 7 | 7 | 11 | 7 | 10 | 5 | | 11 | 4 | 4 |
| 2 | | 10 | 5 | | 2 | | 10 | 2 | | 2 | | 10 | 5 | 5 | 2 | | 10 | 3 | |
| 2 | | 9 | 1 | | | | 9 | 1 | | 2 | | 9 | 3 | | 1 | | 9 | 1 | |
| 1 | | 8 | 3 | | 3 | | 8 | 1 | | 2 | | 8 | 2 | | 3 | | 8 | 2 | |
| 1 | | 7 | 1 | | | | 7 | 1 | | 2 | | 7 | 7 | | 2 | | 7 | 3 | |
| | | 6 | 1 | | | | 6 | | | 2 | | 6 | | | 3 | | 6 | 2 | |
| | | 5 | | | | | 5 | | | | | 5 | 1 | | 1 | | 5 | 4 | |
| | | 4 | | | | | 4 | | | | | 4 | | | | | 4 | | |
| | | 3 | 1 | | | | 3 | | | | | 3 | | | | | 3 | | |
| | | 2 | 1 | | | | 2 | | | | | 2 | | | | | 2 | | |
| | | 1 | | | | | 1 | | | | | 1 | | | | | 1 | | |
| | | 0 | | | | | 0 | | | | | 0 | | | | | 0 | | |
| Ap. M.. | 13 | | 11 | | 14 | | | 14 | | 11 | | | 10 | | 12 | | | 11 | |
| Cor..... | .7 | | .6 | | .7 | | | 0 | | 0 | | | .8 | | 0 | | | .5 | |
| Med.... | 13.7 | | 11.6 | | 14.7 | | | 14 | | 11 | | | 10.8 | | 12 | | | 11.5 | |
| M. D... | 3 | | 2.2 | | 3 | | | 2.6 | | 2.7 | | | 2.3 | | 3.6 | | | 4 | |
| Var..... | .22 | | .19 | | .20 | | | .19 | | .24 | | | .21 | | .30 | | | .35 | |

Ap. M. = apparent median; Cor. = correction; Med. = median; M. D. = median deviation; Var. = Variability

TABLE III (a)

INITIAL COURTIS TEST

Group E

| ADDITION | | | | | SUBTRACTION | | | | MULTIPLICATION | | | | DIVISION | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Atts. | | | Rts. | | Atts. | | | Rts. | Atts. | | | Rts. | Atts. | | | Rts. |
| Frequency | Deviation | Score | Frequency | Deviation | Frequency | Deviation | Score | Frequency | Deviation | Frequency | Deviation | Score | Frequency | Deviation | Score | Frequency | Deviation |
| | | 24 | | | | | 24 | | | | | 24 | | | 24 | | |
| | | 23 | | | | | 23 | | | | | 23 | | | 23 | | |
| | | 22 | | | | | 22 | | | | | 22 | | | 22 | | |
| 1 | | 21 | | | | | 21 | | | | | 21 | | | 21 | | |
| 2 | | 20 | | | 4 | | 20 | 2 | | | | 20 | | | 20 | | |
| | | 19 | | | 2 | | 19 | | | | | 19 | | | 19 | | |
| 1 | | 18 | 2 | | 3 | | 18 | 1 | | | | 18 | 1 | | 18 | | |
| | | 17 | 1 | | | 2 | 17 | 4 | 2 | | | 17 | | | 17 | | |
| 1 | | 16 | | | 4 | 9 | 16 | 1 | 2 | | | 16 | 1 | | 16 | 1 | |
| 4 | 8 | 15 | | | 2 | 6 | 15 | 1 | 1 | | | 15 | 5 | 7 | 15 | 4 | 6 |
| 3 | 5 | 14 | | | 1 | 1 | 14 | 3 | 1 | | | 14 | 2 | 5 | 14 | 3 | 5 |
| | 2 | 13 | 2 | | 4 | | 13 | 3 | 4 | 10 | | 13 | 3 | 6 | 13 | 2 | 5 |
| 7 | 7 | 12 | 4 | 9 | 5 | | 12 | 5 | 3 | 7 | | 12 | 5 | 5 | 12 | 6 | 6 |
| 2 | | 11 | 3 | 7 | 2 | | 11 | 3 | 3 | 3 | | 11 | 3 | | 11 | 3 | |
| 2 | | 10 | 3 | 3 | 4 | | 10 | 2 | 4 | | | 10 | 3 | | 10 | 2 | |
| 4 | | 9 | 4 | | 1 | | 9 | 4 | 6 | | | 9 | 2 | | 9 | 2 | |
| 2 | | 8 | 5 | | | | 8 | | 2 | | | 8 | 4 | | 8 | 4 | |
| 2 | | 7 | 1 | | | | 7 | | 4 | | | 7 | | | 7 | 2 | |
| 1 | | 6 | 3 | | | | 6 | 2 | | | | 6 | 2 | | 6 | | |
| | | 5 | | | | | 5 | | | | | 5 | | | 5 | | |
| | | 4 | 1 | | | | 4 | | | | | 4 | 1 | | 4 | 1 | |
| | | 3 | 3 | | | | 3 | | | | | 3 | | | 3 | 2 | |
| | | 2 | | | | | 2 | | | | | 2 | 1 | | 2 | | |
| | | 1 | | | | | 1 | | | | | 1 | | | 1 | | |
| | | 0 | | | | | 0 | | | | | 0 | | | 0 | | |
| Ap. M.. | 12 | | 10 | | 14 | | | 13 | 11 | | | 8 | 12 | | | 12 | |
| Cor..... | .5 | | 0 | | 0 | | | .3 | .3 | | | .5 | .2 | | | .1 | |
| Med.... | 12.5 | | 10 | | 14 | | | 13.3 | 11.3 | | | 8.5 | 12.2 | | | 12.1 | |
| M. D... | 3.4 | | 2.7 | | 3.5 | | | 3.7 | 2.7 | | | 2.6 | 3.1 | | | 3.2 | |
| Var..... | .27 | | .27 | | .25 | | | .28 | .24 | | | .31 | .25 | | | .26 | |

TABLE IV

FINAL COURTIS TEST

Group T

| ADDITION | | | | | SUBTRACTION | | | | | MULTIPLICATION | | | | | DIVISION | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Atts. | | | Rts. | | Atts. | | | Rts. | | Atts. | | | Rts. | | Atts. | | | Rts. | |
| Frequency | Deviation | Score | Frequency | Deviation | Frequency | Deviation | Score | Frequency | Deviation | Frequency | Deviation | Score | Frequency | Deviation | Frequency | Deviation | Score | Frequency | Deviation |
| 1 | | 24 | | | 1 | | 24 | | | | | 24 | | | | | 24 | | |
| 1 | | 23 | 2 | | 1 | | 23 | | | | | 23 | | | | | 23 | | |
| 1 | | 22 | | | 1 | | 22 | 2 | | | | 22 | | | | | 22 | | |
| | | 21 | | | 2 | | 21 | | | 1 | | 21 | | | | | 21 | | |
| | | 20 | | | | | 20 | 1 | | | | 20 | | | 1 | | 20 | | |
| | | 19 | 1 | | 1 | | 19 | | | | | 19 | | | | | 19 | 1 | |
| | | 18 | | | 2 | 7 | 18 | 2 | | | | 18 | | | 1 | | 18 | | |
| 2 | | 17 | | | 5 | 6 | 17 | 1 | | 5 | | 17 | | | 1 | | 17 | 1 | |
| 3 | | 16 | 1 | 4 | 5 | 5 | 16 | 3 | 10 | 2 | 6 | 16 | 1 | | | | 16 | 1 | |
| 2 | 5 | 15 | 1 | 1 | 1 | | 15 | 3 | 5 | 1 | 4 | 15 | 1 | | 1 | | 15 | 1 | |
| 4 | 6 | 14 | 4 | 6 | 5 | | 14 | 1 | 2 | 5 | 6 | 14 | 4 | | 3 | | 14 | 2 | |
| 6 | 6 | 13 | 1 | 3 | 1 | | 13 | 3 | 3 | 5 | 5 | 13 | 3 | 6 | 3 | 7 | 13 | 3 | 6 |
| 2 | | 12 | 5 | 5 | 3 | | 12 | 1 | | 1 | | 12 | 2 | 7 | 4 | 8 | 12 | 2 | 6 |
| 3 | | 11 | 2 | | | | 11 | 2 | | 3 | | 11 | 4 | 4 | 3 | 3 | 11 | 1 | 4 |
| 2 | | 10 | 2 | | 2 | | 10 | 7 | | 4 | | 10 | 5 | | 4 | | 10 | 4 | 4 |
| 4 | | 9 | | | 1 | | 9 | 1 | | 1 | | 9 | 3 | | 4 | | 9 | 3 | |
| 1 | | 8 | 3 | | 1 | | 8 | 2 | | 1 | | 8 | 1 | | 3 | | 8 | 4 | |
| | | 7 | 6 | | | | 7 | | | 1 | | 7 | 2 | | 3 | | 7 | 3 | |
| | | 6 | 1 | | | | 6 | 1 | | 2 | | 6 | | | 1 | | 6 | 2 | |
| | | 5 | 3 | | | | 5 | | | | | 5 | 2 | | | | 5 | 1 | |
| | | 4 | | | | | 4 | 1 | | | | 4 | 3 | | | | 4 | 2 | |
| | | 3 | | | | | 3 | 1 | | | | 3 | | | | | 3 | 1 | |
| | | 2 | | | | | 2 | | | | | 2 | 1 | | | | 2 | | |
| | | 1 | | | | | 1 | | | | | 1 | | | | | 1 | | |
| | | 0 | | | | | 0 | | | | | 0 | | | | | 0 | | |
| Ap. M.. | 13 | | 12 | | 16 | | | 13 | | 13 | | | 11 | | 11 | | | 10 | |
| Cor..... | .8 | | 0 | | .6 | | | .3 | | .8 | | | 0 | | .6 | | | .3 | |
| Med.... | 13.8 | | 12 | | 16.6 | | | 13.3 | | 13.8 | | | 11 | | 11.6 | | | 10.3 | |
| M. D... | 2 | | 4.5 | | 2.9 | | | 3.7 | | 3.3 | | | 3 | | 2.6 | | | 3.5 | |
| Var..... | .15 | | .37 | | .18 | | | .28 | | .24 | | | .27 | | .22 | | | .34 | |

TABLE IV (a)

FINAL COURTIS TEST

Group E

| ADDITION | | | | | SUBTRACTION | | | | | MULTIPLICATION | | | | | DIVISION | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Atts. | | | Rts. | | Atts. | | | Rts. | | Atts. | | | Rts. | | Atts. | | | Rts. | |
| Frequency | Deviation | Score | Frequency | Deviation | Frequency | Deviation | Score | Frequency | Deviation | Frequency | Deviation | Score | Frequency | Deviation | Frequency | Deviation | Score | Frequency | Deviation |
| | | 24 | | | 4 | | 24 | 1 | | | | 24 | | | 1 | | 24 | | |
| 1 | | 23 | | | 3 | 6 | 23 | 1 | | | | 23 | | | | | 23 | 1 | |
| 1 | | 22 | | | 3 | 6 | 22 | 3 | | 1 | | 22 | | | | | 22 | | |
| | | 21 | 1 | | | 3 | 21 | 2 | | 1 | | 21 | 1 | | 1 | | 21 | | |
| | | 20 | 1 | | | 1 | 20 | 2 | 3 | 1 | | 20 | 1 | | 1 | | 20 | | |
| 3 | | 19 | | | 5 | 5 | 19 | 1 | 3 | 3 | | 19 | | | 2 | | 19 | 2 | |
| 4 | 7 | 18 | | | 1 | | 18 | 1 | 4 | 1 | | 18 | 1 | | 4 | 6 | 18 | 5 | 6 |
| 2 | 6 | 17 | 3 | 9 | 3 | | 17 | 2 | 6 | 1 | 5 | 17 | 1 | | 1 | 3 | 17 | 1 | 3 |
| 3 | 6 | 16 | 3 | 6 | 3 | | 16 | 2 | 2 | 4 | 9 | 16 | 1 | | 8 | 8 | 16 | 3 | 7 |
| 4 | 4 | 15 | 2 | 6 | 3 | | 15 | 4 | 2 | 5 | 5 | 15 | | 6 | 2 | | 15 | 3 | 3 |
| 3 | | 14 | 3 | 3 | 3 | | 14 | 3 | | 5 | | 14 | 2 | 4 | 2 | | 14 | 4 | |
| 4 | | 13 | 4 | | 1 | | 13 | 2 | | 4 | | 13 | 7 | 9 | 2 | | 13 | 2 | |
| 3 | | 12 | 3 | | 1 | | 12 | 1 | | 3 | | 12 | 3 | 3 | 2 | | 12 | 1 | |
| 4 | | 11 | 6 | | | | 11 | 2 | | 3 | | 11 | 2 | | 1 | | 11 | 4 | |
| | | 10 | 5 | | | | 10 | 2 | | | | 10 | 2 | | 2 | | 10 | 1 | |
| | | 9 | 1 | | | | 9 | 2 | | | | 9 | 6 | | 1 | | 9 | 2 | |
| | | 8 | | | | | 8 | 1 | | | | 8 | 3 | | 1 | | 8 | 1 | |
| | | 7 | | | | | 7 | | | | | 7 | 2 | | 1 | | 7 | 2 | |
| | | 6 | | | | | 6 | | | | | 6 | | | | | 6 | | |
| | | 5 | | | | | 5 | | | | | 5 | | | | | 5 | | |
| | | 4 | | | | | 4 | | | | | 4 | | | | | 4 | | |
| | | 3 | | | | | 3 | | | | | 3 | | | | | 3 | | |
| | | 2 | | | | | 2 | | | | | 2 | | | | | 2 | | |
| | | 1 | | | | | 1 | | | | | 1 | | | | | 1 | | |
| | | 0 | | | | | 0 | | | | | 0 | | | | | 0 | | |
| Ap. M.. | 15 | | 14 | | 19 | | | 16 | | 15 | | | 12 | | 16 | | | 15 | |
| Cor..... | .8 | | 0 | | .4 | | | 0 | | .4 | | | .6 | | .4 | | | 0 | |
| Med.... | 15.8 | | 14 | | 19.4 | | | 16 | | 15.4 | | | 12.6 | | 16.4 | | | 15 | |
| M. D... | 3.1 | | 3.2 | | 4.3 | | | 4.6 | | 2.6 | | | 3.1 | | 3 | | | 3.7 | |
| Var..... | .20 | | .23 | | .22 | | | .27 | | .17 | | | .24 | | .18 | | | .25 | |

In the final Courtis Tests we find, Tables IV and IV (a) that the median number of problems attempted by Group E exceeds that of Group T by 2 in addition, 2.8 in subtraction, 1.6 in multiplication and 4.8 in division,—a superiority of 11.2 problems. The median number of problems correctly solved by Group E exceeds that of Group T by 2 problems in addition, 2.7 in subtraction, 1.6 in multiplication and 4.7 in division,—a total superiority of 11 problems. In the final test the variability in the number of "attempts" was greater for Group E by 5% in addition, 4% in subtraction and less by 7% in multiplication and by 4% in division,—an average superiority of 5% for Group E. The variability of the number of "rights" for Group E in the final Courtis Test was less by 14% in addition, 1% in subtraction, 3% in multiplication, and 9% in division,—an average of 6.7% less variability for Group E.

With respect to the number of problems attempted at the close of the eight practice periods, Groups T and E improved 8.5% and 34.4% respectively; with regard to the number of problems correctly solved, Group T lost 2.5% while Group E gained 30.9%. (For summary see Table I.)

Fig. 9 graphically sets forth the relation of the medians and variability as given in Table I. The medians of the initial attempts for all four fundamental operations show very slight differences between Groups T and E. The arrows practically fall within a vertical line. This is not true for the medians of the final attempts and rights. Here the medians for Group E in both attempts and rights exceed that of Group T by 2.8 abscissae points. The median of final "rights" for Group T occupies a position .3 abscissae points farther to the left than its position in the initial test, showing a decrease in accuracy with the increase in speed.

The marked superiority of the economical method is clearly seen in curve a, Fig. 4. The greatest superiority is in division and the least in addition.

Figs. 5-9 show in a striking manner the decrease in variability under the economical method, the graphs becoming more compact around the median. This characteristic is most marked in the addition graphs (Fig. 5), showing decreased variability both in the number of final attempts and in the number of rights for Group E as previously stated. Group E excels Group T at and after the sixth work period (Table II) in addition, at the fifth period and thereafter in subtraction and multiplication, and at the fourth period and thereafter in division. The irregular rates of improvement for

both groups, apparent in the columns of totals (Table II) and in the graphs (Fig. 1), derived from Table II, indicate uncontrollable changes in attitude of pupils probably due to flagging interest and to unfavorable bodily and environmental conditions.

Individual records as well as fluctuations of group averages in the case of Group E indicate that the arithmetical habits acquired in the elementary school, asserted themselves more insistently in the 3rd and 7th work periods. Here we find a marked increase in the average time required for completion of the problems when a decrease should have been expected. Statements of some of the members of the group corroborate the conclusion.

The greatest amount of improvement is found in division of Group E (Table II). Here the average time required to solve the seven problems of the eighth work period was 65 seconds less than the time required in the first work period (213-148). The corresponding improvement for multiplication, subtraction and addition in Group E was 37, 50 and 44 seconds respectively. Improvements for Group T in the same order as given above for Group E were 46, 28, 13 and 28 seconds. The time required to complete the third work period of Group E was 246 seconds. This time exceeds that of any other work period of either group for any of the four fundamental operations. Here the average time was 27 seconds more than in the corresponding period and operation for Group T. This time, (246 seconds), for Group E was 33 seconds more than in the first work period, 69 seconds more than in the second work period and 98 seconds more than in the last work period. These figures are significant in view of the fact that it was at this point that the pupils complained most of the interference of their former habits. Moreover it was in the third period that the economical method was first really used by Group E. The pupils confessed to having used the traditional method during the second period because they found their desire for speed led them, unconsciously, to break away from the economical method and to lapse into the traditional method.

The greater difficulty in learning division and the greater final improvement agrees with results of Chapman.[6] He found that improvement with practice is always greatest in the more complicated processes. Division involves a greater variety of processes than any of the other three fundamental operations.

[6] Chapman, James Crosby, Thesis (Ph.D.), Columbia, 1914.

The irregular character of the learning curves for Group T is probably due to the waxing and waning of interest incident to the continuance of a method devoid of novelty. An effort was made to iron out the irregularity by giving each pupil a copy of his group record for the previous work periods, by calling attention to the achievements of superior pupils and by urging a maximum of effort on the part of all. The same incentives were given to both groups.

The curves in Fig. 2 are composites of curves a, b, c, and d of Fig. 1. Each point in the curves shows the total time required in each work period to complete the problems in addition, subtraction, multiplication and division. The efficiency of Group E, as shown by these curves, unmistakably exceeds that of Group T after the fourth work period. Accuracy is incorporated in the graphs of Figs. 1, 2 and 3 by adding to the group averages the number of seconds which would have been required, at the average speed, to correct all errors. Group T had 9/10 per cent of error and Group E 7/10 per cent for the average of the eight work periods. The total number of errors made by each group during each of the work periods and the average number of errors per pupil are given in Table V and the former is graphically shown in Fig. 10.

TABLE V

TOTAL ERRORS AND AVERAGE ERRORS PER PUPIL

| Groups | Work Periods | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | |
| | T. | Av. | T. | Av. | T. | Av. | T. | Av. | T. | Av. | T. | Av. | T. | Av. | T. | Av. |
| T. | 180 | 5.6 | 157 | 4.9 | 180 | 5.6 | 140 | 4.4 | 147 | 4.6 | 183 | 5.7 | 187 | 5.9 | 191 | 6.0 |
| E. | 173 | 5.4 | 135 | 4.2 | 100 | 3.1 | 112 | 3.5 | 134 | 4.2 | 119 | 3.4 | 152 | 4.7 | 140 | 4.4 |

The curves in Fig. 3 show the efficiency of the six initially fastest, six median and six slowest pupils of both groups. The increase of speed with practice is most conspicuous in curve b, the learning curve for the slowest six pupils of Group E. The difference between the average time in the initial and final work periods is 335 seconds (1086-751 sec.). The median six of this group decreased the average time 235 seconds (846-611), while the fastest six decreased it 165 seconds (595-430). The corresponding decreases in Group

T were 246, 103 and 55 seconds. These differences represent the time saved in the eighth work period as compared with the first.

The percentage of increase in speed was nearly uniform throughout Group E, while only the slowest pupils in Group T made noteworthy improvement with practice; but this improvement was much less than that made by the corresponding six pupils of Group E. The six slowest, the six median and the six fastest pupils of Group E surpassed the corresponding pupils of Group T at the fifth, eighth and fifth work periods respectively. Expressed in per cent, the slowest, median and fastest pupils of Group T improved 23, 13 and 9 per cent respectively while those in Group E improved 31, 28 and 28 per cent.

TABLE VI

| Groups of Six Pupils | Improvement in Seconds | | Improvement in Per Cent | |
|---|---|---|---|---|
| | Group T | Group E | Group T | Group E |
| Slowest | 246 | 335 | 23 | 31 |
| Median | 103 | 235 | 13 | 28 |
| Fastest | 55 | 165 | 9 | 28 |

## Summary and Conclusions

(1) According to the final Courtis Tests the group of pupils working economically shows the following striking superiority over the group working traditionally: (a) in the number of problems *attempted,* 11.2; (b) in the number of problems correctly solved, 1; (c) 6.7% less in variability; (d) with respect to the number of problems attempted at the close of the eighth work period 33.4%. (Group T actually lost 2.5%.)

These statistics gain enormously in importance when it is remembered that (1) the pupils working traditionally showed a decided superiority in the initial Courtis Tests, and (2) that Group E employed the method of Group T for the first of the eight work periods.

(2) Of the four fundamental operations, division shows the greatest absolute improvement in Group T as well as Group E. These values for Groups E and T are 65 and 46 seconds respectively. These results confirm the observations made by Chapman referred to above.

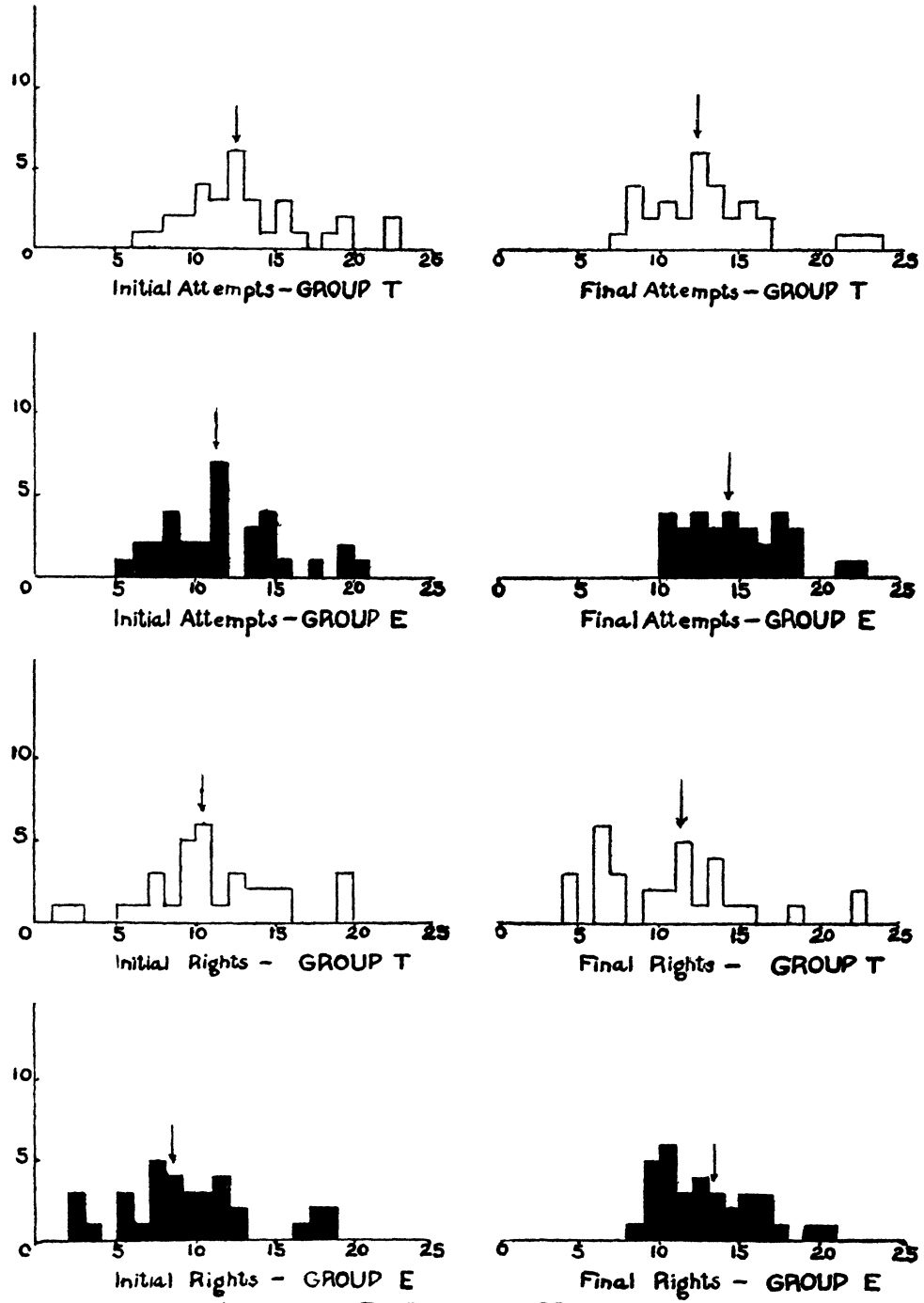


FIG 5, COURTIS TESTS IN ADDITION

ABSCISSAS = PROBLEMS ORDINATES = PUPILS

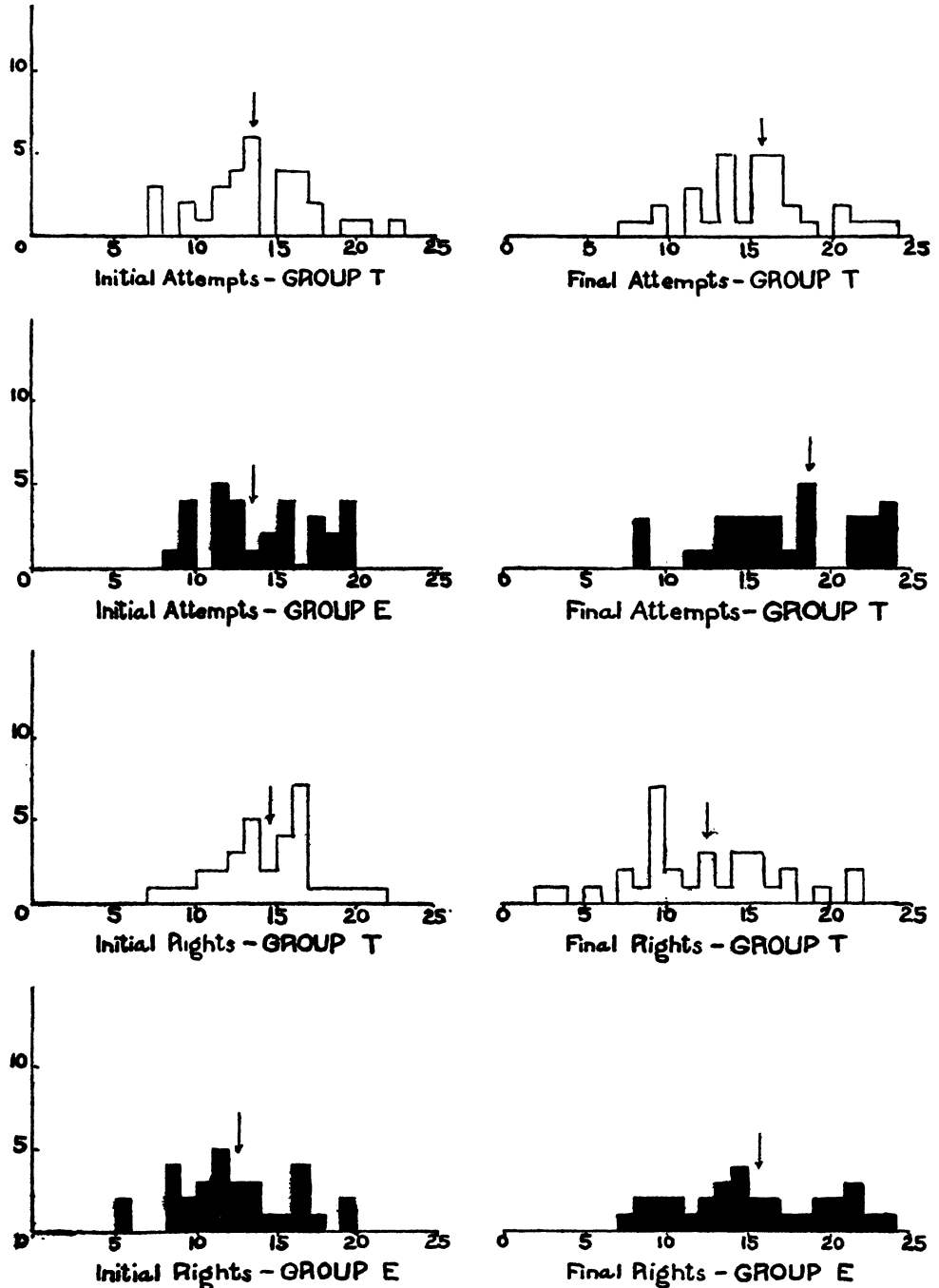


FIG. 6, COURTIS TESTS IN SUBTRACTION

ABSCISSAS = PROBLEMS ORDINATES = PUPILS

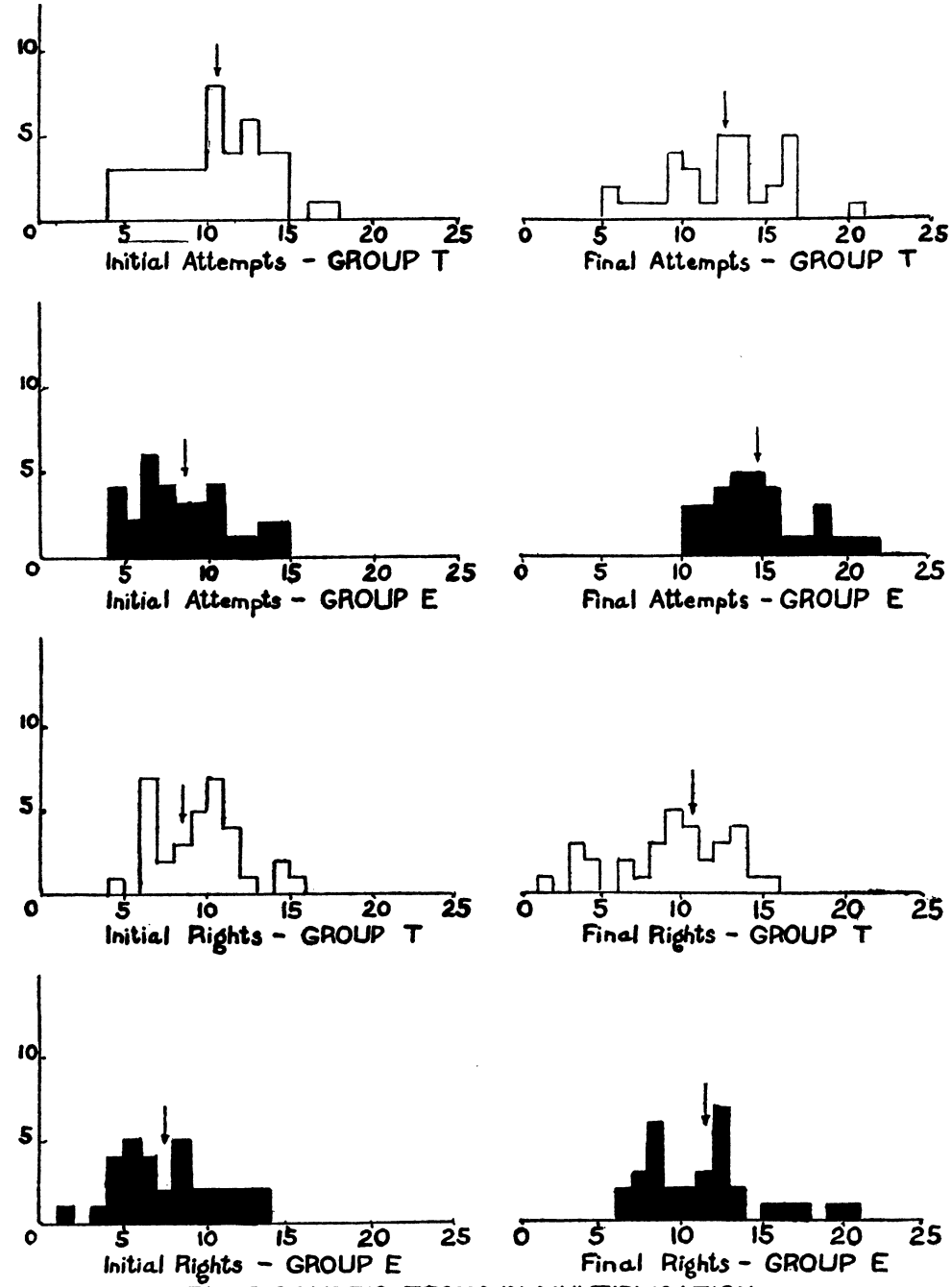


FIG. 7, COURTIS TESTS IN MULTIPLICATION
ABSCISSAS = PROBLEMS ORDINATES = PUPILS

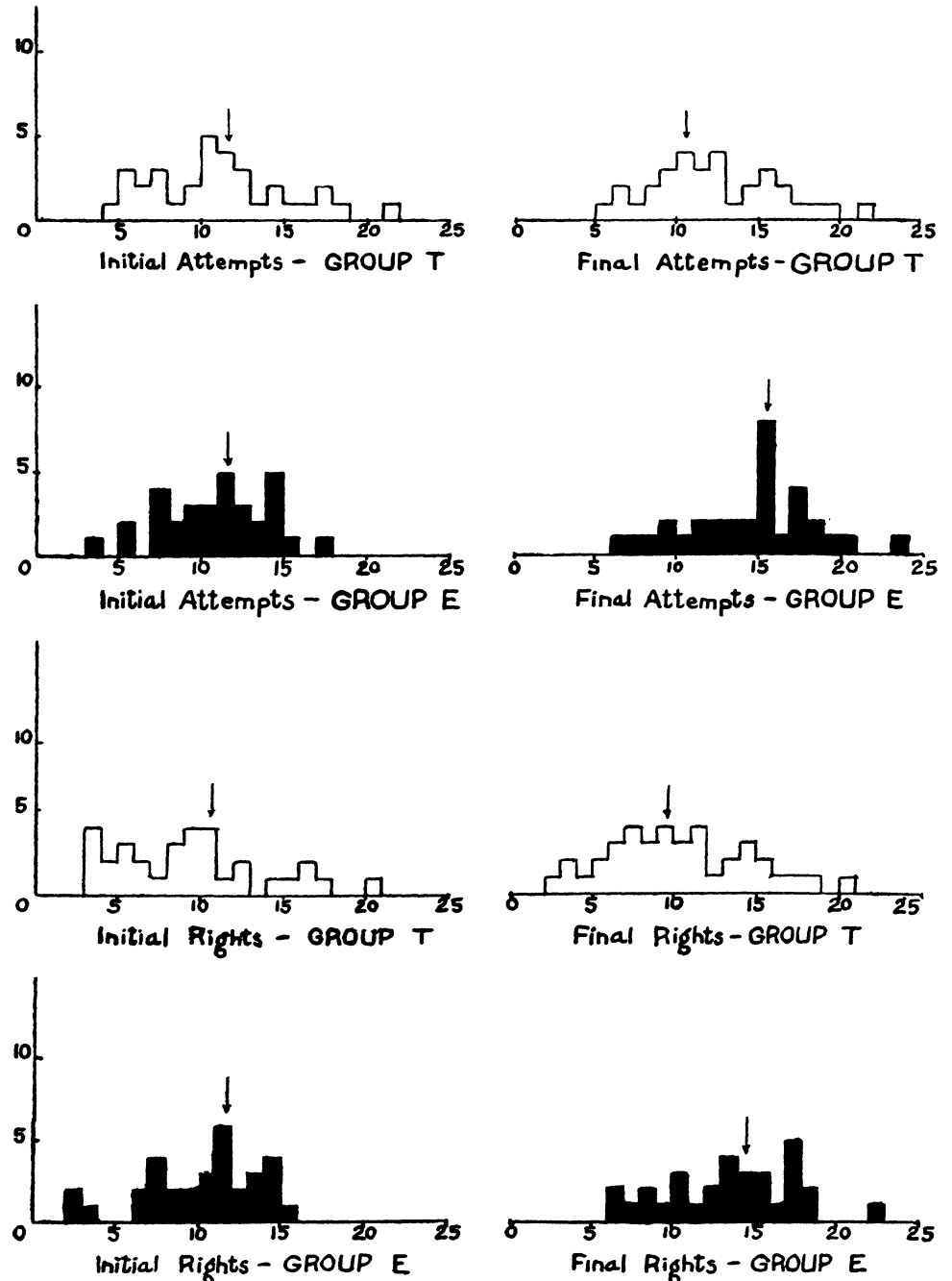


FIG. 8, COURTIS TESTS IN DIVISION

ABSCISSAS = PROBLEMS ORDINATES = PUPILS

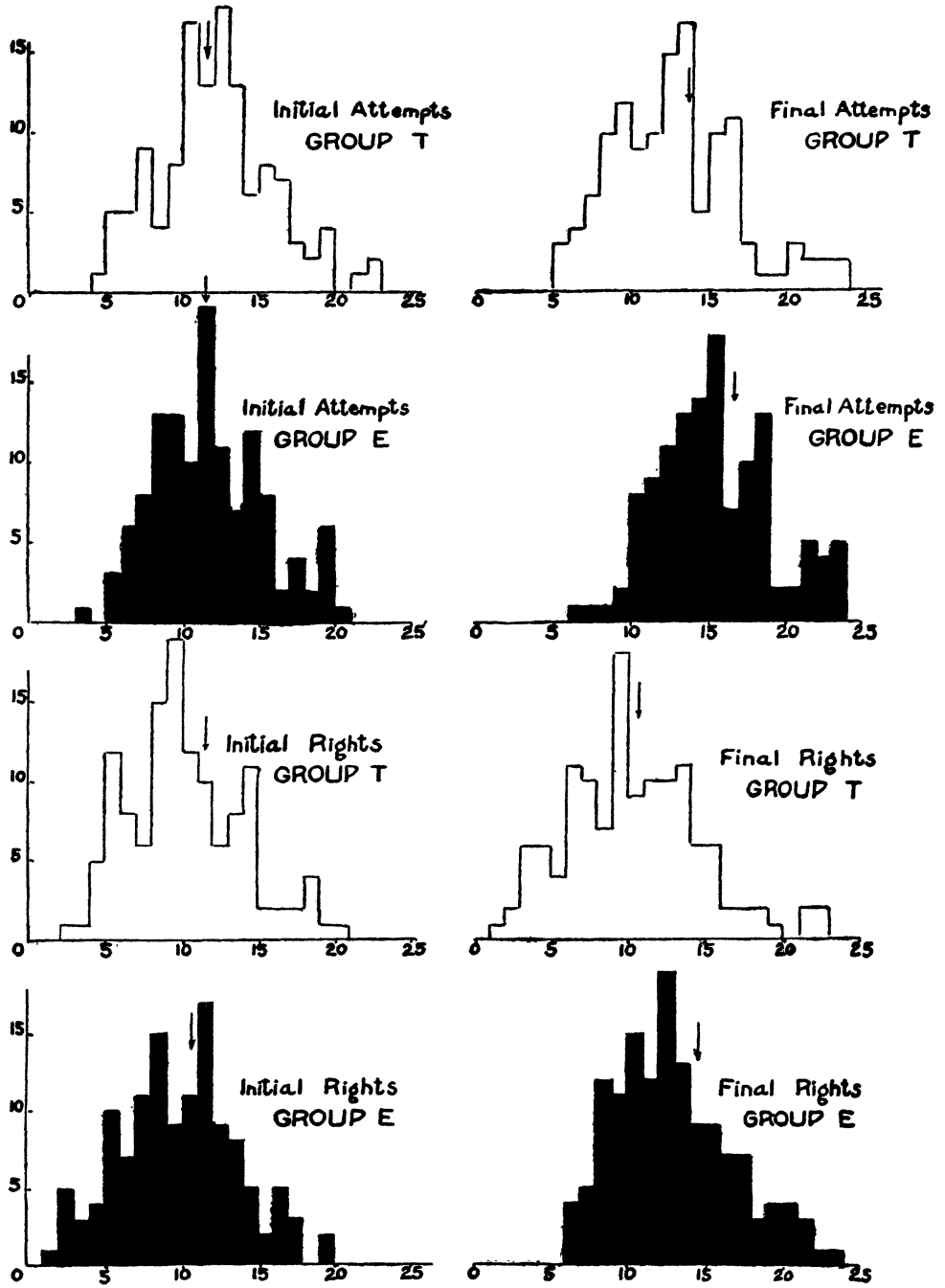


FIG. 9, COURTIS TESTS — Totals for four fundamental operations

(3) Individual records and group averages of Group E indicate that the interference of established habits of work manifest themselves irregularly. The phenomenon of interference was greatest in the third and seventh work periods.

(4) The superiority of the economical group is again shown in the reduction of the amount of time required to solve the problems of the last work period as compared with the first period. In this respect the economical group shows a higher efficiency than Group T by 19, 9, 37 and 16 seconds for division, multiplication, subtraction and addition respectively.

(5) The gain in speed of Group E over the corresponding pupils of Group T in the four fundamental operations combined, of the six slowest, six medians and six fastest pupils, is 90, 132 and 110 seconds respectively.

The adoption by elementary and secondary schools of the method of arithmetical computations employed by Group E would eliminate an important factor of waste and make possible a considerable reduction in the time now devoted to arithmetic. In the four fundamental operations it appears that pupils should be permitted to think in terms of results only and restrict, so far as possible, the audito-motorizing mechanism.

The economical method should be employed as soon as possible to prevent the formation of 'interference' habits and for the reason that the economical method is more favorable to the development of habits of attention.